

Probabilistic Evaluation of Traffic Flux and Bridges Safety

La evaluación probabilística de Flujo del Tráfico y Seguridad del Puentes

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Abstract

The traffic flux is a complex phenomenon that occurs in existing bridges, what makes their dynamic behavior difficult to understand and badly defined. Indeed, all laws and models reflect from idealizations and empirical approximations. For this reason, the present work proposes a methodological approach for a macroscopic modeling of the traffic based on the probabilistic aspect of the phenomenon. The whole system is described using a probabilistic model where all the factors influencing the highway traffic and the safety of bridges are considered. The basic traffic data and results get using this approach lead to describe the crucial phenomena caused by traffic flux which serves to identify the vehicles speeds limits, the appropriate density and the minimum distance between two moving objects on bridges roadways.

Keywords: Flux, Highway traffic, Probabilistic model, Statistic model, Structure safety, Safe distance

Resumen

El flujo de tráfico es un fenómeno complejo que ocurre en los puentes existentes, lo que hace que su comportamiento dinámico sea difícil de entender y esté mal definido. De hecho, todas las leyes y modelos se reflejan a partir de idealizaciones y aproximaciones empíricas. Por esta razón, el presente trabajo propone un enfoque metodológico para una modelización macroscópica del tráfico basada en el aspecto probabilístico del fenómeno. Todo el sistema se describe utilizando un modelo probabilístico donde se consideran todos los factores que influyen en el tráfico de la carretera y la seguridad de los puentes. Los datos y resultados del tráfico básico utilizan este enfoque para describir los fenómenos cruciales causados por el flujo de tráfico que sirve para identificar los límites de velocidad de los vehículos, la densidad adecuada y la distancia mínima entre dos objetos en movimiento en las carreteras de los puentes.

Palabras clave: Flujo, Tráfico de la carretera, Modelo probabilístico, la Estadística de modelos, Estructura de seguridad, Distancia segura

1. Introduction

The daily traffic on bridges increases over time and all over the world, that's why it is vital to find a way to preserve such structures from failure and damages. This can be realized by either enhancing the capacity of the bridge to match the traffic flux or modify authorities' laws regarding the circulation as vehicles running speeds limitation and safe distance increasing (Tuny, 1979).. In both cases a good estimation of traffic flux and traffic effect must be conducted.

In the traffic process, the flux is continually changing as a result of the contribution of different dynamic properties in this phenomenon in addition to the transport tools and the road obstacles influence (barriers, convoys, climatic conditions, etc...). All of the aforementioned conditions make difficult to create an adequate mathematical model that represents the traffic flux.

Thus, many research works have been carried out in the field of traffic flux estimation and evaluation in bridges. Some works regarding the traffic flux characteristics were based on sensors collected data. (Zhao et al., 2009) analyzed the traffic flux characteristics including vehicles speeds and density relationship in Beijing using remote traffic microwave

sensor data. (Lu et al., 2012) proposed a method to estimate traffic speed with single inductive loop event data. Although such site-based sensors are the most common tools to obtain the basic data for the traffic flow characteristic analysis, the obtained results can be affected by the installation position of the traffic sensor which can cause the difference on traffic flows and speeds between the adjacent upstream and downstream sections. Moreover, the installation and maintenance of sensors take a lot of human and financial resources. Further, the conditions of vehicles circulation on bridges were studied by (Enright, 2010) where a data model was established using a normal distribution considering the speeds and the intervals between vehicles.

The work presented herein is an analytical approach for traffic flux estimation based on the probabilistic aspect of the phenomenon. The main purpose is on one hand the prediction of traffic and more generally the completion of traffic measurements over time (prediction) and space (distribution), and on the other hand the study of the structure loading and the determination of the traffic conditions (boundary conditions). That is why statistical methods were used in order to propose adequate and automated solutions that can be modular and adapted to technical and climate conditions that represent the case of the present study

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2. Static Model of Transport Flux

As the circulation regime is changing in time (during a day), the law of its variation cannot be described. That is why obtained results using static approaches are mostly unsuitable for stability calculation to estimate the probability of failure and bridges life time. For this reason, IP (Image Processing) data collected and generalized by (Enright, B., 2010) using special programs serve in this work to calculate serious

unpredictable changes in the UBM (Universal Background Model).

In the present work, the statistical model of vehicles in motion flux proposed below is established and an analytical study of probability of circulation regime changes in time is elaborated. The distribution function of vehicles number of one circulation lane as shown in Fig.1 on a given interval is given based on the following assumptions:

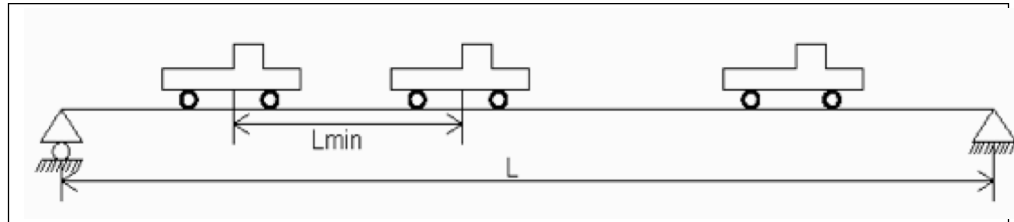


Figure 1. Minimum safe distance on highway bridges (Alessandro Lipari, 2016)

- The velocity v (m/s) and the intensity i (1/h) of vehicles movement are considered constant in the time interval τ . They can vary when they are discretized at each time interval, exemple: 1 hour;
- Overtaking in the bridge is excluded;
- The distance between vehicles (in meters) located in an interval, is considered as the minimum safety distance L_{min} ;
- The influence of cars on each others on the same or opposite lane is not taken into account;
- The situation change in the interval l occurs (discretized) in a time interval equal to $\frac{l}{v}$

The probability $P_j^{-(l)}$ of the number of vehicles j on the interval l is subjected to the Poisson law (Garber N.J. and Lester, A.H., 2014) (Equation 1 and 2):

$$P_j^{-(l)} = cm^j (j!)^{-1} e^{-m} \quad (1)$$

$$C = \left[\sum_{j=0}^n P_j^{-(l)} \right]^{-1} = \left[\sum_{j=0}^n m^j / j! \right]^{-1} e^m \quad (2)$$

Where C is a constant determined from the maximum number of vehicles probabilities $|l / L_{min}|$ which may be found at the same time in the interval l .

The minimum safety distance authorized between the gravity centers of nearby vehicles depends on the speed and intensity of traffic. According to Table 1, where a comparative illustration between safety distance authorized between vehicles running on bridges in the site of Algeria and Russia in the period 2008-2009 is established, we can observe a quite difference in the safe distance related to the traffic conditions difference of Russian and Algerian roads. Moreover, the distance values calculated by the elementary formula provides good results for most speeds between 7-20 m/s as compared to the get values derived from Algerian and Russian roads recorded data in the period 2008-2009.

Table 1: Comparison on safe distance between Algerian and Russian roads according to vehicles running speeds (m) (Get from Guebchman, 1990)

	Speeds (m/s)				
	5	10	15	20	25
	Authorized distance between vehicles (m)				
Values found in the site of Algeria	15	20	28	36	54
Values found in the site of Russia	17	23	35	44	66
Theoretical values	15	30	48	76	100

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Speeds intervals shown in Table 1 are provided in some conditioned measure, they increase with the increasing of loading vehicles size. The intensity is related to the speed by correlation equations obtained using the sample data processing by the least squares method, for a mixed flux we have (Equation 3):

$$v = 16,4 - 4,2 * 10^{-3} u \quad (3)$$

And for trucks flux we have:

$$v = 15 - 3,4 * 10^{-3} u \quad (3.a)$$

Where u is the flux intensity it is represented by the number of vehicles per hour.

Theoretically, the maximum intensity for a single band does not exceed $1000 \text{ l}/\tau$ (Ouleski. B.E. and Bacilev. A.I., 1969).

Practically, the daily limit intensity does not exceed 5000 vehicles / day, and the probable speed of loaded trucks traffic varies between 10 to 20 m / s.

The likelihood of the Poisson hypothesis is verified by the samples of traffic on a single band (lane A) of Algerian road section at different times by day in the present section as detailed below:

During one hour, every second we noted the vehicles number on the intervals lengths 25, 40 and 80 meters respectively with taking into account the complete change in the situation of the vehicles distribution during this time as represented in Fig.2. Then, we defined the traffic intensity and the sample results in addition to the estimators of expected values m^* and dispersion (variance) D^* given in table 2, Equation 4.

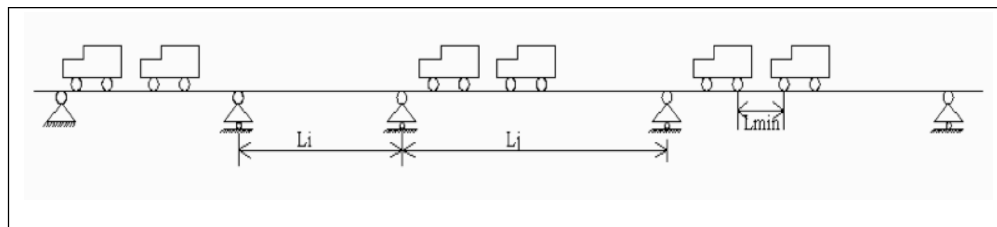


Figure 2. The Position of Vehicles on the Deck and the Minimum Distance between Them

$$m^* = n^{-1} \sum_{i=1}^n n_i * p_i \quad (4)$$

Table 2: Expected and variance values for various flux intensities

U vehicles/h	The track segment length (m)					
	25		40		80	
	m	D	m	D	m	D
400	0.375	0.300	0.387	0.324	0.780	0.860
650	0.570	0.540	0.550	0.510	1.030	1.090

The approach of statistical moment's m^* and D^* corresponding to the defined intensity u in the interval l , reflects the plausibility of the allowed hypothesis by the distribution law: standard poissonier flux of events.

The traffic density which is the number of vehicles realized on a measurement of interval length of a band is $d = \frac{m^*}{l}$ and that can be defined by the empirical linear Equation 5:

$$d = k * v \quad (5)$$

The parameter k is coefficient which; according to (H. Wang, et al, 2009) and (Lum, K., et al., 1998); is around 1,9-3,7 and $10^{-5} \text{ l} / \text{m}$, and according to (Kadiyali, L. R., 1987) data is around 2.9 -3.4 and $10^{-5} \text{ l} / \text{m}$.

For big intensities ($u > 400 \text{ l} / \tau$) the calculation of the probability by the Equation 1 leads to remarkable errors due to the influence of the flux constraint. Therefore, there is no complete change of situation in the interval $\Delta t = \frac{l}{v}$, the effect of stress can be approximately interpreted as the disappearance of a part (the full case) of null events, when there is no vehicle on the given interval, we correct the



allowed law (Equation 1) excluding a part of null event ($j = 0$), by proportionate distribution with the probability $\bar{p}_j^{-(e)}$ for which $j \neq 0$.

For this, we define first the number M of vehicles subjected to constraint during an hour as given in the following Equation 6.

$$M = U - \sum_{j=1}^n j \bar{p}_j^{-(e)} \frac{3600}{l} v \quad (6)$$

So, the number of complementary vehicles, located on the sample, under the same stress (Equation 7):

$$Mj = Mj \bar{p}_j^{-(e)} = \sum_{j=1}^n j \bar{p}_j^{-(e)} \quad (7)$$

The probability $\bar{p}_j^{-(e)}$ for j vehicles ($j \neq 0$) to 'fall' on the interval l that takes into account the correction of stress is defined by the Equation 8:

$$P_j^{(l)} = \bar{P}_j^{(l)} + \bar{P}_j^{(l)} \left[U - \sum_{j=1}^n j \bar{P}_j^{(l)} \frac{3600}{l} v \right] : \left[\frac{3600}{l} v \sum_{j=1}^n j \bar{P}_j^{(l)} \right] \quad (8)$$

The probability of appearance of a null event P_0 will be smaller and is defined by Equation 9:

$$P_0^{(l)} = \bar{P}_0^{(l)} - \sum_{j=1}^n \left[U - \sum_{j=1}^n j \bar{P}_j^{(l)} \frac{3600}{l} v \right] : \left[\frac{3600}{l} v \sum_{j=1}^n j \bar{P}_j^{(l)} \right] \quad (9)$$

3. Estimate of Bridge Loading

The number of events λ_j , for j vehicles at the same time over a length l , for a period $\tau = 1h$ is equal to Equation 10:

$$\lambda_j = \bar{P}_j^{(l)} \frac{3600}{l} v \quad (10)$$

Similarly, we can estimate the number of such events for the bridge employment duration T using a discrete variable of the intensity and the speed of transport movement that vary in time. For such calculation, we can use the cyclograms proposed in (Bando, M. et al. 1998).

The prospect of changes in traffic flux during series of years can be defined mainly on the basis of the extensive evolution particularity of the production of an economic real region (Leurent.F., 1996).. For bridges calculations, only the contribution of heaviest trucks is taken into account, for which thresholds of given reinforcement are achieved in construction.

After the observation of the traffic movement on the considered roadway, it was found that the distribution probability for an infinite column of a particular vehicles type follows a geometric distribution given by the Equation 11:

$$P_j = P^j (1 - P) \quad (11)$$

And the probability of the appearance of vehicles j having the same; type the one after the other; can be defined from the multiplication of probabilities theorem as Equation 12:

$$P_j = P^j \quad (12)$$

With:

P : frequency of appearance of a particular type of vehicle in the column.

In a column, the distribution probability following different types of trucks can be defined by Equation 13:

$$P_j = \prod_{r=1}^j \text{Pr} \quad (13)$$

With:

Pr : probability of appearance of the truck type r in the uniform column.

Distribution, at the same time, of different types of vehicles in the column is defined by Equation 13 and assumed arbitrary. The coefficient μ of vehicle loading represents the relationship between transported change and the total transport column of the vehicle, according to data (Birouli, A.K. and Mikhovitch, C.I., 1978) and (Federov. I.P., 1980), can be equal to 0.4-0.8.

We assume that for all the service period of the bridge, the appearance of a heavy and different loading set is possible and that is on a single band of the road. The number of event Nq , for $q = 1, 2, \dots, Q$, for the q loading calculation during the period T (in years) is estimated by the Equation 14 (taking into account a variation of the intensity and speed).

$$Nq = \sum_{t=1}^T \sum_{s=1}^{12} \sum_{k=1}^{30} \sum_{i=1}^{24} Pq(k, s, i, t) \frac{3600}{l} v \eta^n \quad (14)$$

With:

$$Pq(k, s, i, t) = P_n^{(l)}(k, s, i, t) P_n^{(q)}$$

N : number of cars on the bridge at the same time

$Pq(k, s, i, t)$: Probability of appearance on the bridge for the q loading calculation, this probability of a complex event is defined as the product of probabilities of unfavorable loading combination and calculated using the formula Equation 12 or 13, and the probabilities $P_n^{(l)}(k, s, i, t)$ are calculated by the Equation 8.

k, s, i, t : are values in terms ($k = 1, 2, 3, \dots, 12$), in days ($s = 1, 2, 3, \dots, 30$), in months

($i = 1, 2, 3, \dots, 12$), in years ($t = 1, 2, 3, T$).

The average value of the probability of q loading calculation for the whole period T is calculated by the Equation 15:

$$P_q = \sum_{t=1}^T \sum_{s=1}^{12} \sum_{k=1}^{30} \sum_{i=1}^{24} P_q(k, s, i, t) \quad (15)$$

Here, the numbers 12, 30, 24, correspond respectively to the number of months in the year, days in the month and hours in a day.

In the case when (P_u) and the intensity of movement are given by statistical series, the number of loading calculations N_q is defined by the Equation 16:

$$N_{q1} = T * 12 * 30 * 24 * \frac{3600}{l} * \nu * \mu^n * P_n(q) * \sum_{v=1}^m P(Uv) * P_{n,u}^{(e)} \quad (16)$$

For the crossing movement the probability of intersection of two trucks at a given section of the bridge should theoretically be null. It would be different from 0 if the location of such crossing is given by a certain interval, this probability depends on the relationship that links the length of this interval and the average distances between heavy vehicles of each column. Such situation can be observed in the case of more complex events, for example the intersection of a column of trucks.

The number of events N_q for a movement in both directions will be defined approximately, with an additional element (corrective) by the Equation 17:

$$N_{q2} = \sum_{i=1}^T \sum_{s=1}^{12} \sum_{k=1}^{24} \sum_{t=1}^{30} Pq_1(k, s, i, t) Pq_2(k, s, i, t) \frac{3600}{l} \nu \eta^n \quad (17)$$

In which the probabilities $P_{\omega}(k, s, i, t)$ and $P_{\omega}(k, s, i, t)$ correspond to the different directions of transport movement.

Thus, the appearance of loading calculations for bridges with medium and long spans under normal conditions for mixed movements is studied as a very rare event.

The intensity of the movement on roads for one day is distributed unevenly, according to (Birouli, A.K., 1984) data, it is close to 0 for 1/3 of the day, and can vary from 1.5 to 4 times in the 2/3 of the time remaining. For a high intensity ($u \geq 400$ 1 / h), the passage to a motion with multiple columns is necessary in order to reduce accident situations.

Multiple observations of the circulation (movement) realized by the service operating on different highways in Algeria show a significant decrease in the intensity over time. Table 3 presents an example get from the daily average intensities data (in both directions of motion) for 2 years (2007 and 2008) recorded in economic regions (zones I, II and III). We can observe that the proportion of vehicles over 5T.C in a mixed circulation (movement) is from 18-27% of the total number of vehicles. The proportion of heaviest vehicles (30T.c and more), as reported by (Barrenkov. A.C. and Maltsev. P.I., 1971), is taken equal to 2% of all vehicles.

Table 3: Vehicles percentage in economic zones in the years 2007 and 2008

Convoys of vehicles	Stations of traffic data recording					
	I		II		III	
	2007	2008	2007	2008	2007	2008
light %	16.700	30.800	8.800	8.600	17.300	18.400
Heavy up 2,5 tf %	20.000	10.700	20.000	21.000	26.300	27.600
Heavy up 4,5tf %	31.200	34.900	36.100	32.200	28.500	29.800
Buses %	14.000	4.200	7.900	12.700	6.700	6.000
Heavy vehicles > 5 tf %	18.100	20.400	27.200	25.500	21.200	18.200
Total %	100.000	100.000	100.000	100.000	100.000	100.000
Number of vehicles in 24 h	19500	32700	1200	1550	6150	6800

Further, the viaducts and flyover bridges with special prescription (economic, career) are exploited only by heavy vehicles where the probability and the number of calculations of change can be highly significant.

The transverse distribution of vehicles for roads and bridges with a multi column circulation (movement), according to research, is characterized by the flux division into two parts: heavy vehicles (outside the band) and light (inside the band) (IAURIF, 1997)., holding that the circulation (movement) of light and heavy vehicles is carried out in accordance with the speeds 50-70km / h and 80-110 km / h.

The distribution densities of vehicles constructed according to the crossing widths of bridges have a double modal form for each traffic direction. The important mode corresponds primarily to heavy vehicles (trucks) in a transportation network. The distances between the modes are

between the limits 3m-4m and their distances in edges do not exceed 4m. These data are not practically the distances limits of recommended densities.

Moreover, the transport band on light bridges has almost the same conditions as on bridges having special requirement, which means that the empirical frequency of loading calculations, for them, can be significant. In complex multi-way bridges, the probability P_q and the predictable number of loading calculations N_q are defined as for complex events of k spans loading using the possible number of vehicles n_j and simultaneously with the total absence of loading on the other spans ($s-k$).

From the results obtained through the graphs of Figures 3 and 4, we note that the results are convincing and the influence of the obtained results on the correction depends highly on the speed of traffic and travel time realized. In this context, we note that for a time t less than 0.5



days, the influence of the correction is very important generally for all densities and in particular the densities between 100 and 250 vehicles per hour, but this influence over this period varies in the opposite direction which means that the correction value decreases for the lowest densities mentioned above and even null values (see Figures 3 and 4) and increases for other cases especially for the speed 15 km / h. For this reason, we always try to maintain the traffic flux in urban areas with certain fluidity and a uniform distribution on all circulation roads, as we arrive to guarantee or ensure an adequate velocity according to our traffic plan. In fact, it is a dilemma for our technicians to find an equilibrium between a relatively low speed and a moderate traffic density that are

appropriate to our traffic conditions in the different road networks generally and arches roads networks particularly. Moreover, we are interesting a lot of in rush hours for the traffic organization in order to offer comfort and safety to passengers. Finally, our goal of this modest study is to improve significantly our calculation models and to manage the traffic for the horizons lower than 0.5 days, as we can dare to go further than that like the case of lower horizons where the record time is less than 15.00 minutes (less than 0.25 hours). These results are confirmed when predicting travel time of a traffic flux for two types of bridges and bridges in urban environment where a large crowded city and heavy traffic are preferred.

Table 4: Correction values for the speed 10m / s

Days	0.0	0.1	0.3	1.0	1.3	2.0	2.3	2.6	3.2
U=100 1/h	0.900	0.690	0.650	0.100	0.070	0.010	0.010	0.005	0.000
U=250 1/h	0.680	0.650	0.440	0.250	0.180	0.030	0.020	0.010	0.004
U=500 1/h	0.380	0.380	0.380	0.380	0.350	0.150	0.120	0.090	0.044
U=1000 1/h	0.380	0.380	0.380	0.380	0.370	0.330	0.300	0.270	0.200

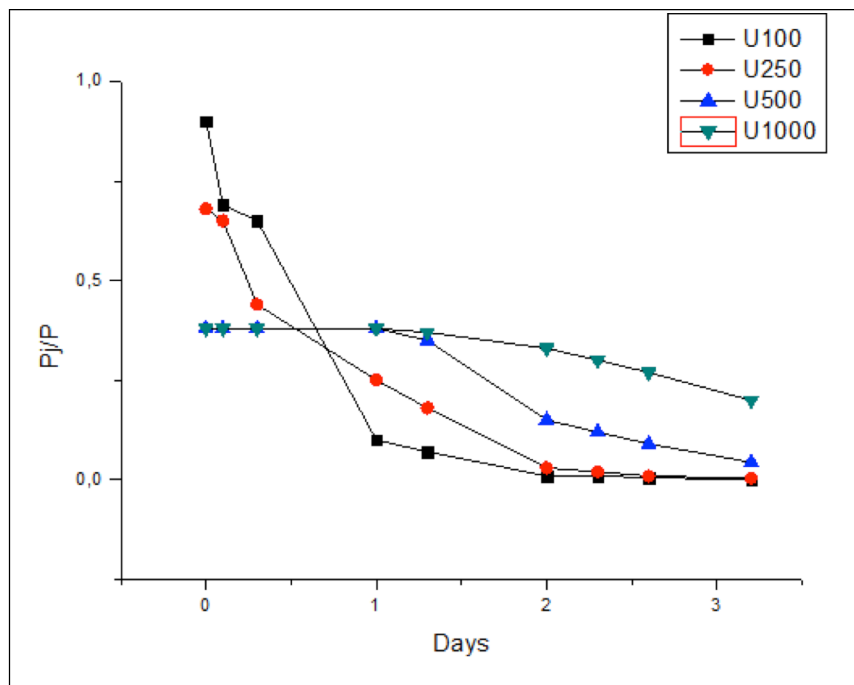


Figure 3. Representative graph of speed influence on the correction values (V=10 m / s)

Table 5: Correction values for the speed 15 m / s

Days	0.0	0.1	0.3	1.0	1.3	1.5	1.8	2.0	2.1
U=100 1/h	0.900	0.800	0.600	0.100	0.070	0.050	0.020	0.010	0.000
U=250 1/h	0.780	0.750	0.610	0.210	0.180	0.140	0.080	0.030	0.020
U=500 1/h	0.580	0.560	0.490	0.360	0.300	0.240	0.180	0.120	0.100
U=1000 1/h	0.040	0.060	0.150	0.250	0.480	0.450	0.390	0.340	0.300

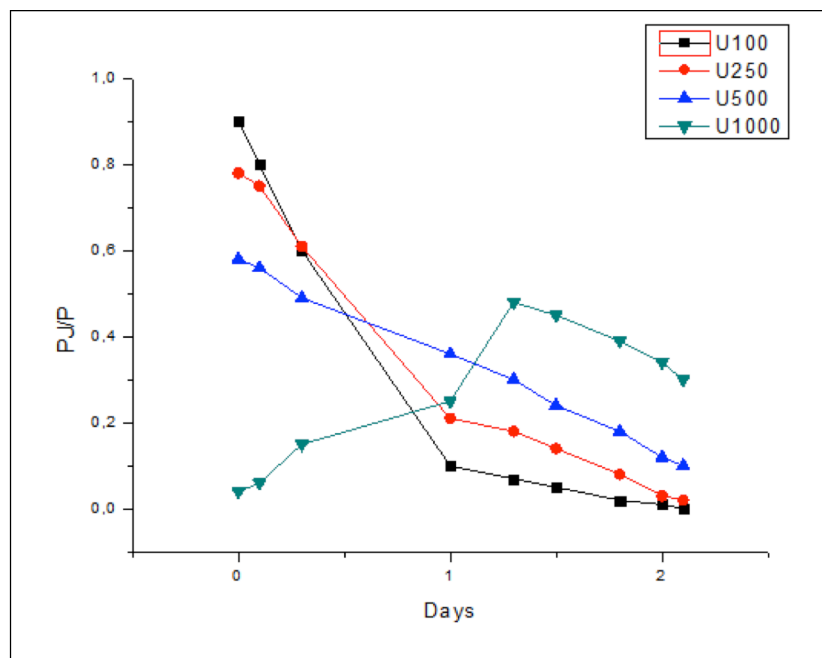


Figure 4. Representative graph of the speed influence on the correction values ($V=15$ m / s)

4. Conclusion

We conclude in this paper that the influence of the correction depends on the intensity and the velocity of vehicles flux and that the minimum safety distance allowed between the gravity centers of neighboring vehicles depends on the speed and the traffic intensity. Indeed, we could understand theoretically and practically for our case study regarding road networks that the traffic flux becomes important for heavy vehicles having a speed between 5 to 10 km / h on Algerian highway bridges decks which represents the case of military and exceptional surcharges as (D240, MC120) and Russians surcharges such as (Hk-80). As a result, the civil surcharges that don't disturb stability of such structures are eventually the systems (A (I), Bc, Bt), which allows to highway bridges to work in reassuring and

confidence intervals and offers a shelter of security for heavy loads, exceptional and unconventional loads.

In the present work, we are convinced that the distribution of traffic flux is difficult to determine where various variables are not considered because of lack of data and availability. In contrast to these physical methods, we propose to estimate directly the causality between the traffic variables and the safety of existing structures in the various circuits of the combined movement and the main trends of the traffic evolution in the city and traffic networks.

Finally, we tried to get some performance by the present method as compared to those obtained with simple process. The future work will develop tests to gather the variables data and study their effects without losing in capacity the predicted traffic flux or the safety of structures in different networks sections.



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